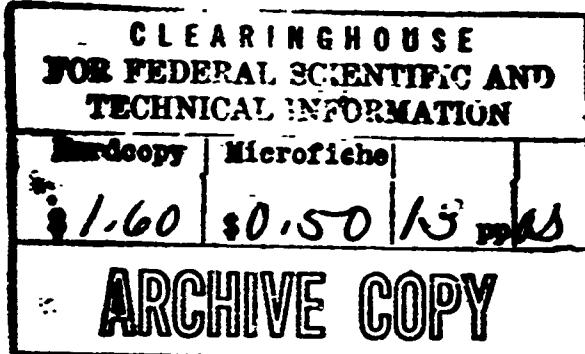


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ISOTOPE EFFECT IN  $\text{Ca}^{40}$ ,  $\text{Ca}^{44}$ , AND  $\text{Ca}^{48}$  CHARGE DISTRIBUTIONS  
FROM 250 MeV ELASTIC ELECTRON SCATTERING\*

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\* Work supported in part by the U. S. Office of Naval Research,  
Contract [Nonr 225(67)], and by the National Science Foundation,  
Contract No. NSF GP 2417.

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Isotope Effect in  $\text{Ca}^{40}$ ,  $\text{Ca}^{44}$ , and  $\text{Ca}^{48}$  Charge Distributions  
from 250 MeV Elastic Electron Scattering\*

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Results are presented on the charge density difference between  $\text{Ca}^{40}$  and  $\text{Ca}^{48}$ . They show that pronounced changes are made in the charge density, that is, in the proton distribution, by the addition of the eight neutrons. The following features, which the earlier investigation of the  $\text{Ca}^{40}$  -  $\text{Ca}^{44}$  difference<sup>1</sup> suggested, are now apparent. The charge radius, i.e., the overall size, increases by an amount comparable to the change in  $A^{1/3}$ . The charge density at the edge of the nucleus (beyond about 4F) does not increase in the manner suggested by such an overall dilatation, and it is even less in  $\text{Ca}^{48}$  than it is in  $\text{Ca}^{40}$ . Comments on the possible origin of this effect, and of other relevant scattering experiments, are made later in this report.

Experimental and computational procedures are very similar to those of reference 1. Briefly, the experimental method is that of elastic electron scattering from isotopically enriched foils of  $\text{Ca}^{40}$  and  $\text{Ca}^{48}$ . The electron beam was supplied by the Mark III linear accelerator. The isotopic purities of the targets were 99.97% ( $\text{Ca}^{40}$ ) and 97.22% ( $\text{Ca}^{48}$ ). The contamination in the latter target was due mainly to  $\text{Ca}^{40}$ , and was corrected for. The thicknesses of the targets were  $529 \text{ mg/cm}^2$  ( $\text{Ca}^{40}$ ) and  $487 \text{ mg/cm}^2$  ( $\text{Ca}^{48}$ ).

The absence of low-lying levels in both isotopes permits us to use poorer energy resolution than in the  $\text{Ca}^{40}$ - $\text{Ca}^{44}$  experiment<sup>1</sup>, and thus a larger slice of the incident accelerator beam. The measurements of the cross section ratios, expressed in the form

$$D(\theta) = [\sigma_{40}(\theta) - \sigma_{48}(\theta)] / [\sigma_{40}(\theta) + \sigma_{48}(\theta)] , \quad (1)$$

can more easily be continued to larger angles. The added information so obtained about the ratio of the differential cross sections contributes to the greater detail with which we can obtain the quantity  $Q(r)$  :

$$Q(r) = 4\pi r^2 [\rho_{40}(r) - \rho_{48}(r)] , \quad (2)$$

compared to the  $\text{Ca}^{40}$ - $\text{Ca}^{44}$  work. As with that work, we assume a given functional form for the  $\text{Ca}^{40}$  charge density, with parameter values determined previously.<sup>1</sup> The same form is assumed to apply to  $\text{Ca}^{48}$ , with altered parameter values.<sup>2</sup> These altered values are determined by making a least-squares fit to the experimentally measured quantity  $D$ , Eq. (1). By varying the values assumed for  $\text{Ca}^{40}$ , we conclude that the fit is insensitive to them and that what is being determined is the difference in parameter values between  $\text{Ca}^{40}$  and  $\text{Ca}^{48}$ . Two different functional

forms, expressed by

$$\rho(r) = \rho_0 [1 + wr^2/c^2] \{ \exp[(r^n - c^n)/z^n] + 1 \}^{-1} \quad (3)$$

with  $n = 1$  (Fermi) and  $n = 2$  (Modified Gaussian), are considered.

The added flexibility of the parameter  $w$  is needed to achieve an acceptable fit to the data, for the  $n = 1$  case, and has been included in both cases for completeness. The main features of the predicted  $\Omega(r)$ , the quantity which we feel has most significance in our results, are to a large extent the same for the two shapes  $n = 1$  and  $n = 2$ , i.e., they are largely independent of the functional form chosen for  $\rho(r)$ .

The experimental results, and the best fits to them according to the above shapes, are shown in Fig. 1. With 32 pieces of data, and three parameters to be fitted, the values of  $\chi^2$  in the two cases were 26( $n = 1$ ) and 30( $n = 2$ ). Without the extra parameter  $w$  in (3), the value of  $\chi^2$  for  $n = 1$  is 61. Additional experimental points taken in a later run with  $Ti^{48}$  are included in the figure (not in the least squares analysis) to verify the consistency of our results. Some points taken at lower energies to examine the behaviour of  $D(\theta)$  at small values of  $q$ , the recoil momentum, and thus to check on our measurements of target thickness, etc., are also shown. These points, actually measured at  $\theta = 32^\circ$ , are plotted at the value of  $\theta$  which at 250 MeV gives the same value of  $q$ . We have verified by our partial wave analysis that this is a satisfactorily accurate procedure. The agreement with the 250 MeV results is very good.

The quantity  $Q(r)$ , expressing the difference in charge distributions between  $\text{Ca}^{40}$  and  $\text{Ca}^{48}$ , is shown in Fig. 2 for the two shapes  $n = 1$ ,  $n = 2$  in Eq. (3). The various parameters associated with the densities are given in the table. The errors quoted there include, besides those indicated in Fig. 1, an overall  $\pm 1\%$  in the vertical scale to allow for uncertainty in our knowledge of the ratio of the target thicknesses. The third parameter  $w$  is small enough that  $\rho(r)$  is flat near the center, and it is thus still possible to define the half-radius  $r_{0.5}$  and the 90%-10% skin thickness  $t$ . Their values are affected by  $w$ , however. We have included the  $\text{Ca}^{44}$  results for comparison. One sees from the table that the change in half-radius  $\Delta r_{0.5}/r_{0.5}$  is  $\sim 4.5\%$ , of comparable magnitude to the change in  $A^{1/3}$  of  $6.2\%$ . The skin thickness changes negatively by a large amount:  $\Delta t/t \sim -11\%$ . But these numbers, while indicating a large effect, do not reveal the interesting feature displayed in Fig. 2, that the edge density of  $\text{Ca}^{48}$  is less than that of  $\text{Ca}^{40}$ . The overall expansion of  $\text{Ca}^{48}$ , expressed by the  $\Delta r_{0.5}/r_{0.5}$  result, is resisted, so far as the extreme edge is concerned, by some effect which pushes the charge distribution even closer in. The negative change of  $\Delta t/t$  in the  $\text{Ca}^{40}-\text{Ca}^{44}$  comparison indicates the same tendency, but to a smaller extent. The net effect is that compared with  $\text{Ca}^{40}$ ,  $\text{Ca}^{48}$  has a concentration of charge right in the region of the nuclear surface. As regards physical magnitude, we note that the total charge moved to this region, i.e., the negative part of  $Q(r)$  in Fig. 2, is about  $0.4e$ . We emphasize that this is obtained as a phenomenological result, with reasonable assumed functional forms for the charge density, and has not involved the use of a nuclear model.

It is plausible, perhaps, to relate the sharper surface of the charge distribution to a more compact, less easily excited nucleus. Our results then imply that Ca<sup>48</sup> is a more stable structure than Ca<sup>40</sup>. This has been concluded previously from the single-particle nature of the Ca<sup>49</sup> spectrum compared to the complex Ca<sup>41</sup> spectrum. The presence in the Ca<sup>40</sup> wave function of appreciable two-particle, two-hole configuration as revealed by pick-up reactions, seems to be saying the same thing.<sup>3</sup>

In connection with associated experiments we note, from the table, that the mu-mesic  $2p \rightarrow 1s$  x-ray difference predicted by our results is negative. This as yet unmeasured quantity would by itself indicate that Ca<sup>48</sup> is smaller than Ca<sup>40</sup>, as regards mean square radii. It will be interesting to have the experimental value, as an additional accurately known datum on these nuclei, and as a confirmation of our conclusions on the tail of the charge density.

For a possible comparison with nuclear scattering analyses, we note some preliminary results obtained with the single-particle shell model.<sup>4</sup> The hypothesis is made that the spatial wave-function of this model gives directly (after inclusion of the proton size) the charge density. The Woods-Saxon potential in which the protons move has necessarily so many parameters that it could be adjusted to yield almost any desired nuclear charge density (proton density). We find that the shortening of the tail of the Ca<sup>48</sup> density can be reproduced by, for example, having a considerably deeper well than in Ca<sup>40</sup> (about 20% deeper) as well as a radius larger by  $A^{1/3}$ . We would expect that so large a change would be apparent in proton and alpha-particle scattering from these nuclei. Conversely, such scattering analyses, made with the object of examining the difference

between the optical potentials of the two nuclei, could place restrictions on the variations allowable in our analysis of the electron scattering with this model. Other information, relating to the position of single-particle energy levels and of admixtures of higher configurations, is very relevant in this regard, of course.

A paper containing the details of our experiment and its analysis will be prepared soon.

We wish to thank Dr. G. L. Rogosa of the U. S. Atomic Energy Commission and Mr. T. H. Kobisk of Oak Ridge National Laboratory for their help in obtaining the target materials for this experiment. We are grateful to the accelerator crew of the High Energy Physics Laboratory for their cooperation.

## References

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1. R. Hofstadter, G. K. Nöldeke, K. J. van Oostrum, L. R. Suelzle, M. R. Yearian, B. C. Clark, R. Herman, and D. G. Ravenhall, Phys. Rev. Letters 15, 758 (1965). This paper contains references to previous work of direct relevance.
2. Because of the similarity in the excited states of these two nuclei, we hope that any contributions to the scattering from virtual nuclear excitation will cancel in our analysis, because we measure the difference between the nuclei. The only theoretical development in this direction since reference 1 that we are aware of is due to Rawitscher [G. Rawitscher, preprint], who allows for the effect of real excitation in depleting the elastic flux by including an imaginary part in the charge density. We are currently examining this suggestion.
3. One of us (D.G.R.) acknowledges with thanks a stimulating conversation with E. Kashy and many very helpful discussions with P. Axel on these points.
4. J. M. McKinley, L. V. Mather, and D. G. Ravenhall, unpublished calculations.

**Table 1:** Isotopic changes in the parameters  $c$ ,  $z$  and  $w$  of the charge distributions [see Eq. (3)] of  $\text{Ca}^{48}$  and  $\text{Ca}^{44}$  compared with  $\text{Ca}^{40}$ , and related quantities. The values for  $\text{Ca}^{40}$  are taken to be<sup>1</sup>  $c = 3.602 \text{ F}$ ,  $z = 0.576 \text{ F}$ ,  $w = 0$ , for  $n = 1$  (Fermi);  $c = 3.373 \text{ F}$ ,  $z = 2.20 \text{ F}$ ,  $w = 0$  for  $n = 2$  (Modified Gaussian). Errors in  $\Delta c/c$ ,  $\Delta z/z$  and  $\Delta w$  for the  $\text{Ca}^{40}$ - $\text{Ca}^{48}$  comparison have maximum excursions of  $\pm 0.51\%$ ,  $\pm 1.33\%$  and  $\pm 0.20$  respectively, but are correlated. Consequently the errors on the derived quantities  $\Delta r_{0.5}/r_{0.5}$  and  $\Delta t/t$ , where  $r_{0.5}$  is the true half-radius and  $t$  the 90%-10% skin thickness, are somewhat smaller, being about  $\pm 0.1\%$  and  $\pm 1\%$  respectively.  $\Delta E$  is the change predicted in the mu-mesic  $2p \rightarrow 1s$  x-ray energy. The  $\text{Ca}^{40}$ - $\text{Ca}^{44}$  results are from reference 1, where only two parameter fits were made, so that  $\Delta w$  was not obtained.

Comparison	$\Delta c/c$ (%)	$\Delta z/z$ (%)	$\Delta w$	$\Delta r_{0.5}/r_{0.5}$ (%)	$\Delta t/t$ (%)	$\Delta E$ (keV)
$\text{Ca}^{40}$ - $\text{Ca}^{48}$ , $n = 1$	1.94	-8.31	0.079	4.1	-12.4	-0.52
$\text{Ca}^{40}$ - $\text{Ca}^{48}$ , $n = 2$	6.21	-5.53	0.023	4.9	-10.9	-0.48
$\text{Ca}^{40}$ - $\text{Ca}^{44}$ , $n = 1$	2.18	-1.66	--	2.16	-1.58	+0.70

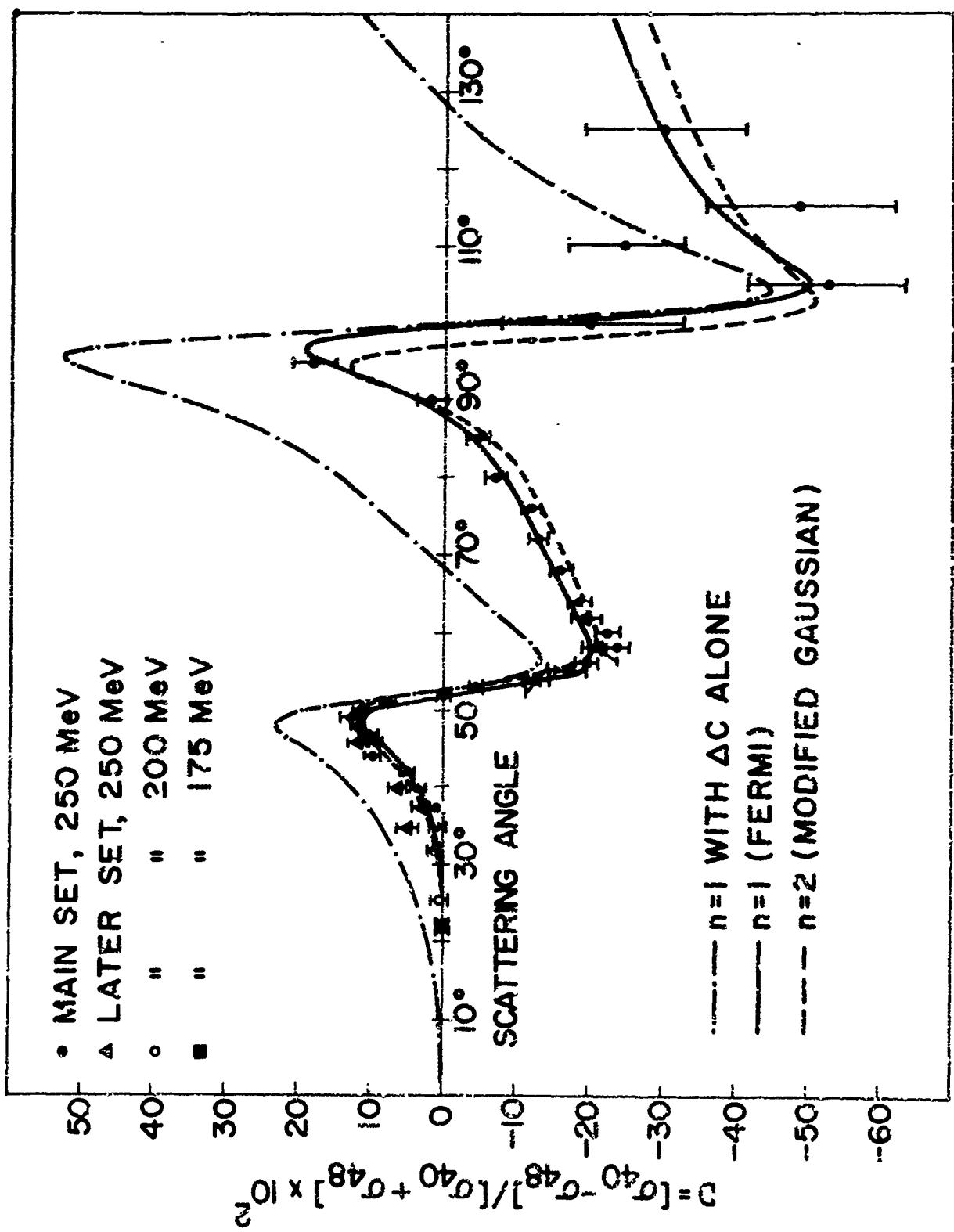


FIGURE 1

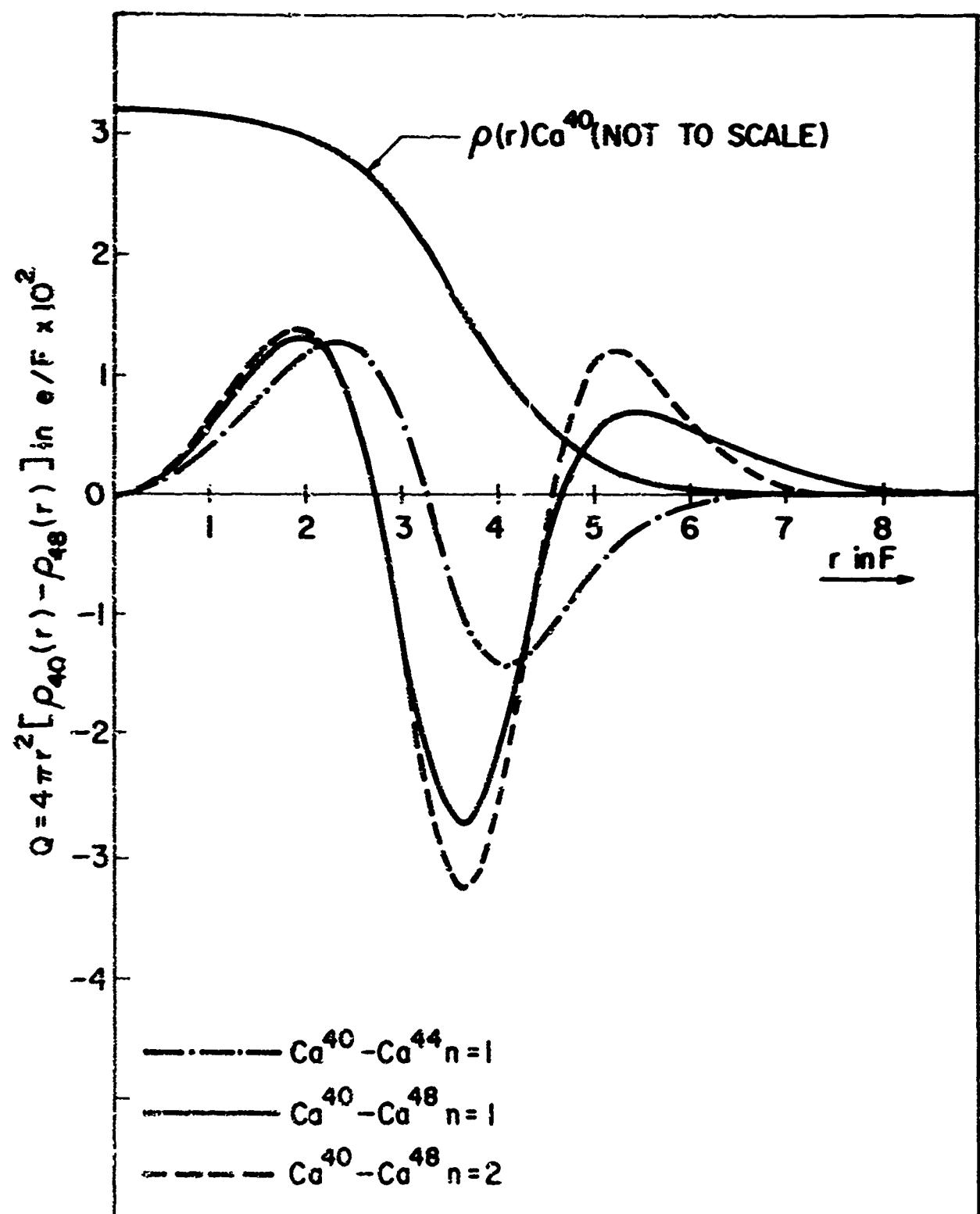


FIGURE 2

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<p>The cross section for elastic electron scattering from an enriched <math>\text{Ca}^{48}</math> target is measured in comparison with the one for <math>\text{Ca}^{40}</math>. The angular dependence of the ratio of these two cross sections is fitted by a curve calculated by phase shift analysis. The best fitting values for the changes in the parameters defining the charge distributions of these nuclei lead to the conclusion that the half-way radius increases by 4.1% and the skin thickness decreases by 12.4%, under the assumption that the charge distribution of both nuclei follow the so-called parabolic Fermi model.</p>		